ROLL NUMBER				

SET	A

INDIAN SCHOOL MUSCAT
FINAL EXAMINATION 2022
MATHEMATICS (041)

CLASS: XII
DATE: 26-11-2022

TIME ALLOTED : 3 HRS.
MAXIMUM MARKS: 80

GENERAL INSTRUCTIONS:

- All questions are compulsory.
- This question paper consists of 38 questions divided into five sections A, B, C, D and E.
- Section A comprises of 18 MCQ of one mark each (from Q01 - 18) and Assertion-Reasoning based questions (from Q19 - Q20)
- Section B comprises of 05 questions of two marks each (from Q21 - 25).
- Section C comprises of 06 questions of three marks each (from Q26 - 31).
- Section D comprises 03 Case-study based questions (from Q32 - 34).
- Section E comprises of 04 questions of five marks each (from Q35 - 38).
- There is no overall choice. However, internal choice has been provided in some questions. You must attempt only one of the alternatives in all such questions.

SECTION A
(Question numbers 01 to 20 carry 1 mark each)

1. Let the function f : $N \to N$ *be defined by* $f(x) = 2x + 3, \forall\, x \in N$. *Then f is*______

   a. Not onto b. bijective c. many-one d. none of these

2. If the area of triangle with vertices (3, 2), (-1, 4) and (6, k) is 7 sq. units then the possible values of k is/are

   a. 3 b. -4 c. -3, 4 d. 3, -4

3. If A= $\begin{bmatrix} 1 & 2 & x \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$ and B= $\begin{bmatrix} 1 & -2 & y \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$ and AB= I, then x + y

   a. 0 b. -1 c. 2 d. None

4. The value of $\int e^x \sec x\,(1 + \tan x)\; dx$ is

   a. $e^x \cos x + C$ b. $e^x \sec x + C$ c. $e^x \sin x + C$ d. $e^x \tan x + C$

5. Write the number of points where f(x) = |x + 2| + |x – 3| is not differentiable.

a. 2　　b. 3　　c. 0　　d. 1

6. Derivative of the function f(x) = $sin(x^2)$

a. $2\cos(x^2)$　　b. $2x\cos(x^2)$　　c. $2x^2\sin(x)$　　d. $2\cos(x)$

7. If y = $A e^{5x} + Be^{-5x}$ $then,$ $\frac{d^2y}{dx^2}$ is equal to

a. 25y　　b. 5y　　c. -25y　　d. 15y

8. The derivative of sin x with respect to log x, is

a. cos x　　b. x cos x　　c. $\frac{x}{\cos x}$　　d. $\frac{\cos x}{x}$

9. The side of an equilateral triangle is increasing at the rate of 0.5 cm/s. Find the rate of increase of its perimeter.

a. 1.5 cm/s　　b. 0.5 cm/s　　c. 3 cm/s　　d. 15cm/s

10. The total revenue received from the sale of x souvenirs in connection with "PEACE DAY" is given by $R(x) = 3x^2 + 40x + 10$. The marginal revenue when 100 souvenirs sold is

a. 34010　　b. 3401　　c. 6410　　d. 640

11. If $\frac{d}{dx}f(x) = g(x)$,then antiderivative of g(x) is

a. f(x)　　b. g(x)　　c. $\frac{1}{2}[f(x)]^2$　　d. $\frac{1}{2}[g(x)]^2$

12. $\int \frac{\sin\sqrt{x}}{\sqrt{x}}\,dx$ is equal to

a. $\cos\sqrt{x} + C$　　b. $2\cos\sqrt{x} + C$　　c. $-2\cos\sqrt{x} + C$　　d. $\sqrt{x}\cos\sqrt{x} + C$

13. If $\int \frac{1}{\sqrt{4-9x^2}}\,dx = \frac{1}{3}\sin^{-1}(kx) + C$, then the value of k is

a. 2　　b. 4　　c. $\frac{3}{2}$　　d. $\frac{2}{3}$

14. If $\int_0^a 3x^2\,dx = 8, then\ the\ value\ of\ a\ is$

a. 2　　b. 3　　c. 4　　d. 8

15. The area of the region bounded by the curve y = x^2 and the line y = 4 is

a. $\frac{32}{2}$ sq. units　　b. $\frac{32}{3}$ sq. units　　c. $\frac{3}{2}$ sq. units　　d. $\frac{32}{5}$ sq. units

16. If A is a square matrix such that $A^2 = A, then, (I + A)^2 - 3A$ is

a. I b. 2A c. 3I d. A

17. If $\begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}\begin{bmatrix} 3 & 1 \\ 2 & 5 \end{bmatrix} = \begin{bmatrix} 7 & 11 \\ k & 23 \end{bmatrix}$, then the value of k is

a. 14 b. 15 c. 16 d. 17

18. A vector in the direction of vector $\hat{\imath} - 2\,\hat{\jmath} + 2\hat{k}$ that has magnitude 15 is

a. $\frac{\hat{\imath}-2\,\hat{\jmath}+2\hat{k}}{3}$ b. $15\hat{\imath} - 30\hat{\jmath} + 30\hat{k}$ c. $\hat{\imath} - 2\hat{\jmath} + 15\hat{k}$ d. $5\hat{\imath} - 10\hat{\jmath} + 10\hat{k}$

Following are Assertion-Reasoning based questions (from Q19 - Q20):
Read the following statements carefully to mark the correct option out of the options given below.
(a) Assertion is true, Reasoning is true; Reasoning is a correct explanation for Assertion.
(b) Assertion is true, Reasoning is true; Reasoning is not a correct explanation for Assertion.
(c) Assertion is true, Reasoning is false.
(d) Assertion is false, Reasoning is true.

19. **Assertion:** To define the inverse of the function f (x) = tan x, any of the intervals $\left(\frac{-3\pi}{2}, \frac{-\pi}{2}\right)$ $\left(\frac{-\pi}{2}, \frac{\pi}{2}\right)$ $\left(\frac{\pi}{2}, \frac{3\pi}{2}\right)$ can be chosen.
Reason: The branch having range $\left(\frac{-\pi}{2}, \frac{\pi}{2}\right)$ is called principal value branch of the function $g(x) = \tan^{-1} x$.

20. **Assertion:** The pair of lines given by $\vec{r} = \hat{\imath} - \hat{\jmath} + \lambda(\hat{\imath} + \hat{\jmath} - \hat{k})$ and $\vec{r} = 2\hat{\imath} - \hat{k} + \mu(\hat{\imath} + \hat{\jmath} - \hat{k})$ are parallel.
Reason: Two lines are parallel if the shortest distance is 0.

SECTION B
(Question numbers 21 to 25 carries 2 marks each)
Very short answer questions

21. For what value of 'k' is the following function continuous at x = 2?

$$f(x) = \begin{cases} 2x + 1\,; x < 2 \\ k \quad ; x = 2 \\ 3x - 1\,; x > 2 \end{cases}$$

22. The radius of an air bubble is increasing at the rate of $\frac{1}{2}$ cm/s. At what rate is the volume of the bubble increasing when the radius is 1 cm?

OR

Show that the function f given by $f(x) = x^3 - 3x^2 + 4x$, $x \in R$ is increasing on R.

23. If matrix $\begin{bmatrix} -2 & x-y & 5 \\ 1 & b & 4 \\ x+y & z & 7 \end{bmatrix}$ is symmetric. Find the values of x, y, z and b.

OR

Find x, if $\begin{bmatrix} x & -5 & -1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 2 \\ 0 & 2 & 1 \\ 2 & 0 & 3 \end{bmatrix} \begin{bmatrix} x \\ 4 \\ 1 \end{bmatrix} = 0$

24. Evaluate: $\begin{vmatrix} 2+\sqrt{3} & 3-\sqrt{2} \\ 3+\sqrt{2} & 2-\sqrt{3} \end{vmatrix}$

25. Show that the points A $(-2\hat{\imath} + 3\,\hat{\jmath} + 5\hat{k})$, B $(\hat{\imath} + 2\,\hat{\jmath} + 3\hat{k})$ and C $(7\hat{\imath} - \hat{k}\,)$ are collinear.

SECTION C
(Question numbers 26 to 31 carries 3 marks each)
Short answer questions

26. Find the value of $tan^{-1}\left(tan\frac{2\pi}{3}\right) + sin^{-1}\left(\sin\frac{2\pi}{3}\right)$

27. If $y = (tan^{-1}x)^2, show\ that\ (x^2+1)^2 y_2 + 2x\,(x^2+1)y_1 = 2$

OR

If $x^m\, y^n = (x+y)^{m+n}, prove\ that\ \frac{d^2y}{dx^2} = 0.$

28. Evaluate $\int_0^{\pi} \frac{x\tan x}{\sec x + \tan x}\, dx$

OR

Evaluate $\int_{-1}^{2} |x^3 - x|\, dx$

29. Find $\int x\, sin^{-1}x\, dx$

30. Find $\int \frac{(x^2+x+1)}{(x+2)(x^2+1)}\, dx$

31. Find a vector which is perpendicular to both and $\vec{a}$ and $\vec{b}$ and $\vec{c}.\vec{d} = 15$ where $\vec{a} = \hat{\imath} + 4\,\hat{\jmath} + 2\hat{k}, \vec{b} = 3\,\hat{\imath} - 2\,\hat{\jmath} + 7\hat{k}$ and $\vec{c} = 2\hat{\imath} - \hat{\jmath} + 4\hat{k}$.

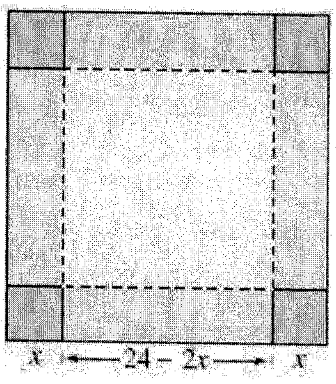
OR

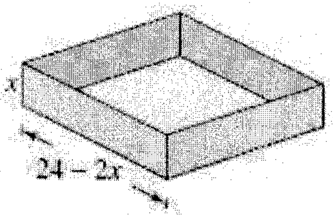
The two skew lines L_1 and L_2 are such that the Line L_1 passes through the point A(1, 2, 1) and its direction is along the vector $\vec{b}_1 = \hat{\imath} - \hat{\jmath} + \hat{k}$, while the Line L_2 passes through the point B(2, -1, -1) and its direction is along the vector $\vec{b}_2 = 2\hat{\imath} + \hat{\jmath} + 2\hat{k}$. Find the shortest distance between the two skew lines L_1 and L_2.

Section D

This section contains three Case-study based questions (from Q32 - Q34).

32. A man has an expensive square shape piece of golden board of size 24 cm. It is to be made into a box without top by cutting a square from each corner and folding the flaps to form a box.





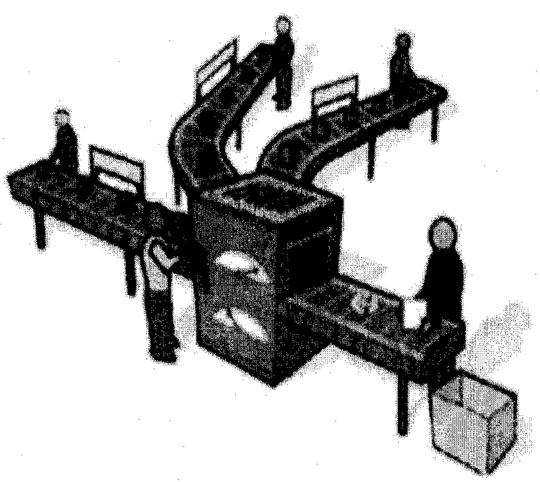
Based on the given information, answer the following questions.

(i) Write an expression for the volume of the open box in terms of x.

(ii) In the first derivative test, if $f'(x)$ changes its sign from positive to negative as x increases through c, then function attains ....at c. (Fill in the blank)

(iii) What should be the side of the square piece to be cut off from each corner of the board so that the volume is maximum?

33. A factory produces three products every day. Their production on a particular day is 45 tons. It is found that production of third product exceeds the production of first product by 8 tons while the total production of first and third product is twice the production of second product. Let x, y and z be the production(in tons) of the first , second and the third product respectively.



Based on the given information, answer the following questions.

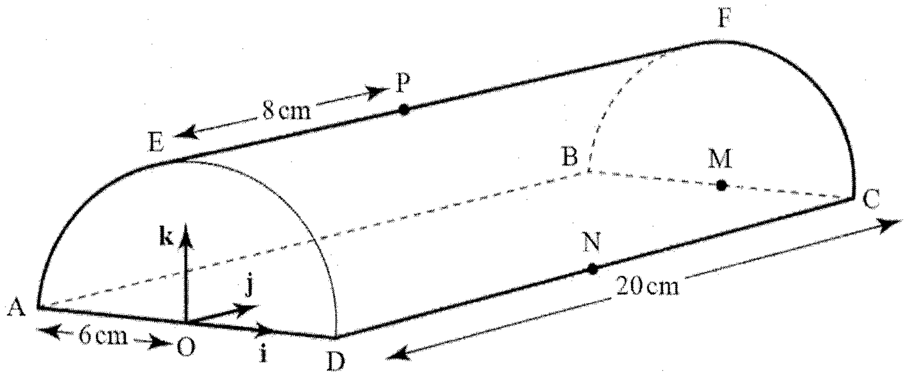
(i) Write the equations in terms of x, y and z and express it in the matrix form AX = B

(ii) Find A^{-1}

OR

(ii) How much is the production of each product?

34. The diagram shows a semi-circular prism with horizontal rectangular base ABCD. The vertical ends AED and BFC are semi-circles of radius 6 cm. The length of the prism is 20 cm. The mid-point of AD is the origin O, the midpoint of BC is M and the midpoint of DC is N. The points E and F are the highest points of the semi-circular ends of the prism. The point P lies on EF such that EP = 8 cm.



Coordinates of points A, D, P and M are (-6,0,0), (6,0,0), (0,8,6) and (0,20,0) respectively.
Unit vectors $\hat{\imath}, \hat{\jmath}$ *and* $\hat{k}$ are parallel to OD, OM and OE respectively.

Based on the given information, answer the following questions.
(i) Express each of the vectors $\overrightarrow{PA}$ and $\overrightarrow{PN}$ in terms of $\hat{\imath}, \hat{\jmath}$ *and* $\hat{k}$.
(ii) Calculate angle APN
(iii) Find a vector perpendicular to $\overrightarrow{PN}$ and $\overrightarrow{PM}$

OR

(iii) Find area of triangle MPN

Section E

(Question numbers 35 to 38 carries 5 marks each)

35. Show that $R = \{(a, b): a, b \in A, | a - b| \text{ is divisible by } 4\}$ where $A = \{x \in Z : 0 \le x \le 12\}$ is an equivalence relation. Find the set of all elements related to 1. Also write the equivalence class [2].

36. Find the area bounded by the curve $x^2 = 4y$ and the line $x = 4y - 2$.

OR

Using integration find the area of region bounded by the triangle whose vertices are (-1, 0), (1, 3) and (3, 2).

37. Find the image of the point (1, 6, 3) in the line $\frac{x}{1} = \frac{y-1}{2} = \frac{z-2}{3}$

OR

Find the equation of the line passing through the point (-1, 3, -2) and perpendicular to each of the lines $\frac{x}{1} = \frac{y}{2} = \frac{z}{3}$ *and* $\frac{x+2}{-3} = \frac{y-1}{2} = \frac{z+1}{5}$.

38. Solve the following linear programming problem graphically:
Maximise: $Z = 2x + 5y$ subject to the constraints: $x \ge 0, y \ge 0, 3x + y \le 6, 2x + 4y \le 8$ *and* $x + y \le 4$. Also write the x and y coordinates of the point, at which Z is maximum.

******END OF THE QUESTION PAPER******

ROLL NUMBER				

SET	B

INDIAN SCHOOL MUSCAT
FINAL EXAMINATION 2022
MATHEMATICS (041)

CLASS: XII
DATE: 26-11-2022

TIME ALLOTED : 3 HRS.
MAXIMUM MARKS: 80

GENERAL INSTRUCTIONS:

- All questions are compulsory.
- This question paper consists of 38 questions divided into five sections A, B, C, D and E.
- Section A comprises of 18 MCQ of one mark each (from Q01 - 18) and Assertion-Reasoning based questions (from Q19 - Q20)
- Section B comprises of 05 questions of two marks each (from Q21 - 25).
- Section C comprises of 06 questions of three marks each (from Q26 - 31).
- Section D comprises 03 Case-study based questions (from Q32 - 34).
- Section E comprises of 04 questions of five marks each (from Q35 - 38).
- There is no overall choice. However, internal choice has been provided in some of the questions. You must attempt only one of the alternatives in all such questions.

SECTION A
(Question numbers 01 to 20 carry 1 mark each)
Multiple choice questions. Select the correct options (from Q01 – Q18)

1. If y = A $e^{5x} + Be^{-5x}$ *then*, $\frac{d^2y}{dx^2}$ is equal to

 a. 25y   b. 5y   c. -25y   d. 15y

2. If the area of triangle with vertices (3, 2), (-1, 4) and (6, k) is 7 sq. units then the possible values of k is\are

 a. 3   b. -4   c. -3, 4   d. 3, -4

3. If P = $\begin{bmatrix} 1 & 2 & a \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$ and Q = $\begin{bmatrix} 1 & -2 & b \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$ and PQ = I, then a + b

 a. 0   b. -1   c. 2   d. None

4. The value of $\int e^x \sec x\,(1 + \tan x)\ dx$ is

 a. $e^x \cos x + C$ b. $e^x \sec x + C$ c. $e^x \sin x + C$ d. $e^x \tan x + C$

5. The function f(x) = |x| is

a. Continuous and differentiable everywhere
b. Continuous and differentiable nowhere
c. Continuous everywhere but differentiable everywhere except 0
d. Continuous everywhere but differentiable nowhere

6. Derivative of the function f(x) = $cos(x^2)$

a. 2 $\cos(x^2)$ b. -2x sin (x^2) c. 2 x^2 sin(x) d. 2 cos (x)

7. Let the function f : N → N be defined by f(x) = 2x + 3, ∀ x ∈ N. Then f is_______

a. Not onto b. bijective c. Many one d. none of these

8. The derivative of cos x with respect to log x, is

a. sin x b. -x sin x c. $\frac{x}{\sin x}$ d. $\frac{\cos x}{x}$

9. The side of an equilateral triangle is increasing at the rate of 0.7 cm/s. Find the rate of increase of its perimeter.

a. 2.1 cm/s b. 0.2 cm/s c. 3 cm/s d. 21 cm/s

10. The total revenue received from the sale of x souvenirs in connection with "PEACE DAY" is given by R(x) = $3x^2 + 40x + 10$. The marginal revenue when 500 souvenirs sold is

a. 340 b. 3040 c. 6410 d. 640

11. If $\frac{d}{dx}g(x) = f(x)$,then antiderivative of f (x) is

a. g(x) b. f(x) c. $\frac{1}{2}[g(x)]^2$ d. $\frac{1}{2}[f(x)]^2$

12. $\int \frac{sin\sqrt{x}}{\sqrt{x}}\, dx$ is equal to

a. cos $\sqrt{x}$ + C b. 2 cos $\sqrt{x}$ + C c. -2 cos $\sqrt{x}$ + C d. $\sqrt{x}\, cos\sqrt{x}$ + C

13. If $\int \frac{1}{\sqrt{4-9x^2}}\, dx = \frac{1}{3}\, sin^{-1}$(kx) + C, then the value of k is

a. 2 b. 4 c. $\frac{3}{2}$ d. $\frac{2}{3}$

14. If $\int_0^a 3x^2\, dx = 8, then\ the\ value\ of\ a\ is$

a. 2 b. 3 c. 4 d. 8

15. The area of the region bounded by the curve y = x^2 and the line y = 4 is

a. $\frac{32}{2}$ sq. units b. $\frac{32}{3}$ sq. units c. $\frac{3}{2}$ sq. units d. $\frac{32}{5}$ sq. units

16. If A is a square matrix such that $A^2 = A$, then, $(I + A)^2 - 3A$ is

a. I b. 2A c. 3I d. A

17. If $\begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}\begin{bmatrix} 3 & 1 \\ 2 & 5 \end{bmatrix} = \begin{bmatrix} 7 & 11 \\ k & 23 \end{bmatrix}$, then the value of k is

a. 14 b. 15 c. 16 d. 17

18. A vector in the direction of vector $2\hat{\imath} - \hat{\jmath} + 2\hat{k}$ that has magnitude 12 is

a. $\frac{2\hat{\imath} - \hat{\jmath} + 2\hat{k}}{3}$ b. $8\hat{\imath} - 4\hat{\jmath} + 8\hat{k}$ c. $\hat{\imath} - 2\hat{\jmath} + 15\hat{k}$ d. $5\hat{\imath} - 10\hat{\jmath} + 10\hat{k}$

Following are Assertion-Reasoning based questions (from Q19 - Q20):
Read the following statements carefully to mark the correct option out of the options given below.
(a) Assertion is true, Reasoning is true; Reasoning is a correct explanation for Assertion.
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(c) Assertion is true, Reasoning is false.
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19. **Assertion:** The pair of lines given by $\vec{r} = \hat{\imath} - \hat{\jmath} + \lambda(\hat{\imath} + \hat{\jmath} - \hat{k})$ and $\vec{r} = 2\hat{\imath} - \hat{k} + \mu(\hat{\imath} + \hat{\jmath} - \hat{k})$ are parallel.
Reason: Two lines are parallel if the shortest distance is 0.

20. **Assertion:** To define the inverse of the function f (x) = tan x, any of the intervals $\left(\frac{-3\pi}{2}, \frac{-\pi}{2}\right)$ $\left(\frac{-\pi}{2}, \frac{\pi}{2}\right)$ $\left(\frac{\pi}{2}, \frac{3\pi}{2}\right)$ can be chosen.
Reason: The branch having range $\left(\frac{-\pi}{2}, \frac{\pi}{2}\right)$ is called principal value branch of the function $g(x) = \tan^{-1} x$.

SECTION B
(Question numbers 21 to 25 carries 2 marks each)

21. Evaluate: $\begin{vmatrix} 2+\sqrt{3} & 3-\sqrt{2} \\ 3+\sqrt{2} & 2-\sqrt{3} \end{vmatrix}$

22. Show that the points A $(-2\hat{\imath} + 3\hat{\jmath} + 5\hat{k})$, B $(\hat{\imath} + 2\hat{\jmath} + 3\hat{k})$ and C $(7\hat{\imath} - \hat{k})$ are collinear.

23. If matrix $\begin{bmatrix} -2 & x-y & 5 \\ 1 & b & 4 \\ x+y & z & 7 \end{bmatrix}$ is symmetric. Find the values of x, y, z and b.

OR

Find x, if $[x \quad -5 \quad -1]\begin{bmatrix}1 & 0 & 2\\0 & 2 & 1\\2 & 0 & 3\end{bmatrix}\begin{bmatrix}x\\4\\1\end{bmatrix} = 0$

24. For what value of 'k' is the following function continuous at x = 2?

$$f(x) = \begin{cases} 2x+1 ; x < 2 \\ k \quad ; x = 2 \\ 3x-1 ; x > 2 \end{cases}$$

25. The radius of an air bubble is increasing at the rate of $\frac{1}{2}$ cm/s. At what rate is the volume of the bubble increasing when the radius is 1 cm?

OR

Show that the function f given by $f(x) = x^3 - 3x^2 + 4x$, $x \in R$ is increasing on R.

SECTION C

(Question numbers 26 to 31 carries 3 marks each)

26. Find the value of $tan^{-1}(1) + cos^{-1}\left(\frac{-1}{2}\right) + sin^{-1}\left(\frac{-1}{2}\right)$

27. Find $\int \frac{dx}{3x^2+13\,x-10}$

28. Find $\int x\, sin^{-1}x\ dx$

29. If $y = (tan^{-1}x)^2, show\ that\ (x^2+1)^2y_2 + 2x\,(x^2+1)y_1 = 2$

OR

If $x^m\,y^n = (x+y)^{m+n}, prove\ that\ \frac{d^2y}{dx^2} = 0.$

30. Evaluate $\int_0^{\pi} \frac{x\tan x}{\sec x+\tan x}\ d$

OR

Evaluate $\int_{-1}^{2} |x^3 - x|\ dx$

31. Find a vector which is perpendicular to both and $\vec{a}$ and $\vec{b}$ and $\vec{c}.\vec{d} = 15$.
If $\vec{a} = \hat{\imath} + 4\hat{\jmath} + 2\hat{k}, \vec{b} = 3\hat{\imath} - 2\hat{\jmath} + 7\hat{k}$ and $\vec{c} = 2\hat{\imath} - \hat{\jmath} + 4\hat{k}$.

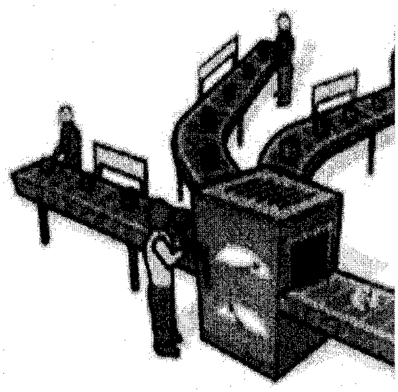
OR

The two skew lines L_1 and L_2 such that Line L_1 passes through the point A(1, 2, 1) and its direction is along the vector $\vec{b_1} = \hat{\imath} - \hat{\jmath} + \hat{k}$, while Line L_2 passes through the point B(2, -1, -1) and its direction is along the vector $\vec{b_2} = 2\hat{\imath} + \hat{\jmath} + 2\hat{k}$. Find the shortest distance between the two skew lines L_1 and L_2.

Section D

This section contains three Case-study based questions (from Q32 - Q34).

32. A factory produces three products every day. Their production on a particular day is 45 tons. It is found that production of third product exceeds the production of first product by 8 tons while the total production of first and third product is twice the production of second product. Let x, y and z be the production(in tons) of the first , second and the third product respectively.



Based on the given information, answer the following questions.

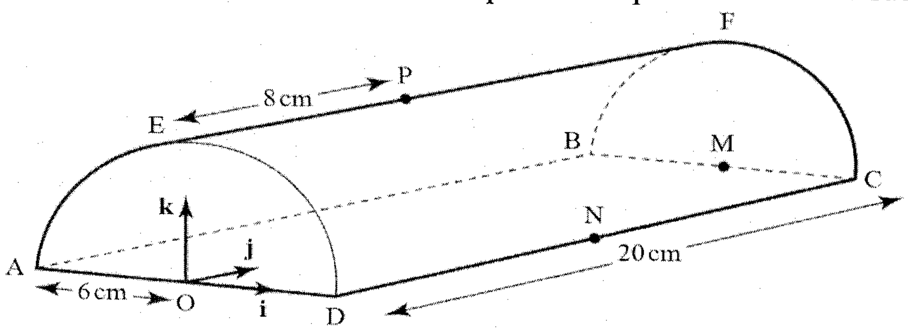
(i) Write the equations in terms of x, y and z and express it in the matrix form AX = B

(ii) Find A^{-1}

OR

(ii) How much is the production of each product?

33. The diagram shows a semi-circular prism with horizontal rectangular base ABCD. The vertical ends AED and BFC are semi-circles of radius 6 cm. The length of the prism is 20 cm. The mid-point of AD is the origin O, the midpoint of BC is M and the midpoint of DC is N. The points E and F are the highest points of the semi-circular ends of the prism. The point P lies on EF such that EP = 8 cm.



Coordinates of points A, D, P and M are (-6,0,0), (6,0,0), (0, 8, 6) and (0,20,0) respectively. Unit vectors $\hat{\imath}, \hat{\jmath}$ *and* $\hat{k}$ are parallel to OD, OM and OE respectively.

Based on the given information, answer the following questions.

(i) Express each of the vectors $\overrightarrow{PA}$ and $\overrightarrow{PN}$ in terms of $\hat{\imath}, \hat{\jmath}$ *and* $\hat{k}$.

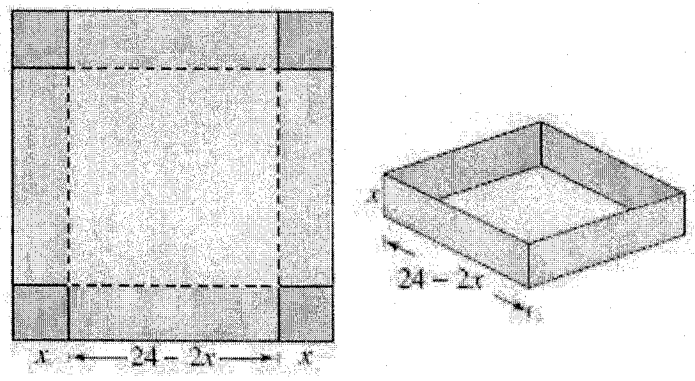
(ii) Calculate angle APN

(iii) Find a vector perpendicular to $\overrightarrow{PN}$ and $\overrightarrow{PM}$

OR

(iii) Find area of triangle MPN

34. A man has an expensive square shape piece of golden board of size 24 cm. It is to be made into a box without top by cutting a square from each corner and folding the flaps to form a box.



Based on the given information, answer the following questions.

(i) Write an expression for the volume of the open box.

(ii) In the first derivative test, if $f'(x)$ changes its sign from positive to negative as x increases through c, then function attains .... at c. (Fill in the blanks)

(iii) What should be the side of the square piece to be cut off from each corner of the board so that the volume is maximum?

Section E

(Question numbers 35 to 38 carries 5 marks each)

35. Solve the following linear programming problem graphically:

Maximise: Z = 5x + 2y subject to the constraints: $x \geq 0$, $y \geq 0$, $x - 2y \leq 2$, $3x + 2y \leq 12$ and $-3x + 2y \leq 3$. Also write the x and y coordinates of the point, at which Z is maximum.

36. Find the area bounded by the curve $x^2 = 4y$ and the line $x = 4y - 2$.

OR

Using integration find the area of region bounded by the triangle whose vertices are (-1, 0), (1, 3) and (3, 2).

37. Show that $R = \{(a, b): a, b \in A, |a - b| \text{ is divisible by } 4\}$ where $A = \{x \in Z : 0 \leq x \leq 12\}$ is an equivalence relation. Find the set of all elements related to 1. Also write the equivalence class [2].

38. Find the image of the point (1, 6, 3) in the line $\frac{x}{1} = \frac{y-1}{2} = \frac{z-2}{3}$

OR

Find the equation of the line passing through the point (-1, 3, -2) and perpendicular to each of the lines $\frac{x}{1} = \frac{y}{2} = \frac{z}{3}$ and $\frac{x+2}{-3} = \frac{y-1}{2} = \frac{z+1}{5}$.

****END OF THE QUESTION PAPER****

ROLL NUMBER				

SET	C

INDIAN SCHOOL MUSCAT
FINAL EXAMINATION 2022
MATHEMATICS (041)

CLASS: XII
DATE: 26-11-2022

TIME ALLOTED : 3 HRS.
MAXIMUM MARKS: 80

GENERAL INSTRUCTIONS:

- All questions are compulsory.
- This question paper consists of 38 questions divided into five sections A, B, C, D and E.
- Section A comprises of 18 MCQ of one mark each (from Q01 - 18) and Assertion-Reasoning based questions (from Q19 - Q20)
- Section B comprises of 05 questions of two marks each (from Q21 - 25).
- Section C comprises of 06 questions of three marks each (from Q26 - 31).
- Section D comprises 03 Case-study based questions (from Q32 - 34).
- Section E comprises of 04 questions of five marks each (from Q35 - 38).
- There is no overall choice. However, internal choice has been provided in some of the questions. You must attempt only one of the alternatives in all such questions.

SECTION A

(Question numbers 01 to 20 carry 1 mark each)

Multiple choice questions. Select the correct options (from Q01 – Q18)

1. The function f(x) = |x| is

   a. Continuous and differentiable everywhere
   b. Continuous and differentiable nowhere
   c. Continuous everywhere but differentiable everywhere except 0
   d. Continuous everywhere but differentiable nowhere

2. If the area of triangle with vertices (3, 2), (-1, 4) and (6, k) is 7 sq. units then the possible values of k is\are

   a. 3 b. -4 c. -3, 4 d. 3, -4

3. If A= $\begin{bmatrix} 1 & 2 & x \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$ and B= $\begin{bmatrix} 1 & -2 & y \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$ and AB= I, then x + y

   a. 0 b. -1 c. 2 d. None

4. The value of $\int e^x \sec x\,(1+\tan x)\ dx$ is

a. $e^x \cos x + C$ b. $e^x \sec x + C$ c. $e^x \sin x + C$ d. $e^x \tan x + C$

5. Let the function f: $N \rightarrow N$ be defined by $f(x) = 2x + 3, \forall\, x \in N$. Then f is_______

a. Not onto b. bijective c. many-one d. none of these

6. Derivative of the function f(x) = $sin(x^2)$

a. $2\cos(x^2)$ b. $2x\cos(x^2)$ c. $2x^2 \sin(x)$ d. $2\cos(x)$

7. If $y = A e^{7x} + Be^{-7x}$ then, $\frac{d^2y}{dx^2}$ is equal to

a. 14y b. 7y c. -7y d. 49y

8. The derivative of sin x with respect to log x, is

a. cos x b. x cos x c. $\frac{x}{\cos x}$ d. $\frac{\cos x}{x}$

9. The side of an equilateral triangle is increasing at the rate of 0.5 cm/s. Find the rate of increase of its perimeter.

a. 1.5 cm/s b. 0.5 cm/s c. 3 cm/s d. 15cm/s

10. The total revenue received from the sale of x souvenirs in connection with "PEACE DAY" is given by $R(x) = 3x^2 + 40x + 10$. The marginal revenue when 100 souvenirs sold is

a. 34010 b. 3401 c. 6410 d. 640

11. If $\frac{d}{dx} f(x) = g(x)$, then antiderivative of g(x) is

a. f(x) b. g(x) c. $\frac{1}{2}[f(x)]^2$ d. $\frac{1}{2}[g(x)]^2$

12. $\int \frac{\sin\sqrt{x}}{\sqrt{x}}\, dx$ is equal to

a. $\cos\sqrt{x} + C$ b. $2\cos\sqrt{x} + C$ c. $-2\cos\sqrt{x} + C$ d. $\sqrt{x}\cos\sqrt{x} + C$

13. If $\int \frac{1}{\sqrt{4-9x^2}}\, dx = \frac{1}{3}\sin^{-1}(kx) + C$, then the value of k is

a. 2 b. 4 c. $\frac{3}{2}$ d. $\frac{2}{3}$

14. If $\int_0^a 3x^2\, dx = 8$, then the value of a is

a. 2 b. 3 c. 4 d. 8

15. The area of the region bounded by the curve y = x^2 and the line y = 4 is

a. $\frac{32}{2}$ sq. units b. $\frac{32}{3}$ sq. units c. $\frac{3}{2}$ sq. units d. $\frac{32}{5}$ sq. units

16. If A is a square matrix such that $A^2 = A, then, (I + A)^2 - 3A$ is

a. I b. 2A c. 3I d. A

17. If $\begin{bmatrix}1 & 2\\3 & 4\end{bmatrix}\begin{bmatrix}3 & 1\\2 & 5\end{bmatrix} = \begin{bmatrix}7 & 11\\k & 23\end{bmatrix}$, then the value of k is

a. 14 b. 15 c. 16 d. 17

18. A vector in the direction of vector $\hat{\imath} - 2\hat{\jmath} + 2\hat{k}$ that has magnitude 6 is

a. $\frac{\hat{\imath}-2\hat{\jmath}+2\hat{k}}{3}$ b. $2(\hat{\imath} - 2\hat{\jmath} + 2\hat{k})$ c. $\hat{\imath} - 2\hat{\jmath} + 15\hat{k}$ d. $5\hat{\imath} - 10\hat{\jmath} + 10\hat{k}$

Following are Assertion-Reasoning based questions (from Q19 - Q20):
Read the following statements carefully to mark the correct option out of the options given below.
(a) Assertion is true, Reasoning is true; Reasoning is a correct explanation for Assertion.
(b) Assertion is true, Reasoning is true; Reasoning is not a correct explanation for Assertion.
(c) Assertion is true, Reasoning is false.
(d) Assertion is false, Reasoning is true.

19. **Assertion:** To define the inverse of the function f (x) = tan x, any of the intervals $\left(\frac{-3\pi}{2}, \frac{-\pi}{2}\right)$ $\left(\frac{-\pi}{2}, \frac{\pi}{2}\right)$ $\left(\frac{\pi}{2}, \frac{3\pi}{2}\right)$ can be chosen.
Reason: The branch having range $\left(\frac{-\pi}{2}, \frac{\pi}{2}\right)$ is called principal value branch of the function g(x) = $\tan^{-1}$ x.

20. **Assertion:** The pair of lines given by $\vec{r} = \hat{\imath} - \hat{\jmath} + \lambda(\hat{\imath} + \hat{\jmath} - \hat{k})$ and $\vec{r} = 2\hat{\imath} - \hat{k} + \mu(\hat{\imath} + \hat{\jmath} - \hat{k})$ are parallel.
Reason: Two lines are parallel if the shortest distance is 0.

SECTION B
(Question numbers 21 to 25 carries 2 marks each)
Very short answer questions

21. If matrix $\begin{bmatrix}-2 & x-y & 5\\1 & b & 4\\x+y & z & 7\end{bmatrix}$ is symmetric. Find the values of x, y, z and b.

OR

Find x, if $[x \quad -5 \quad -1]\begin{bmatrix}1 & 0 & 2\\0 & 2 & 1\\2 & 0 & 3\end{bmatrix}\begin{bmatrix}x\\4\\1\end{bmatrix} = 0$

22. Show that the points A $(-2\hat{\imath} + 3\hat{\jmath} + 5\hat{k})$, B $(\hat{\imath} + 2\hat{\jmath} + 3\hat{k})$ and C $(7\hat{\imath} - \hat{k})$ are collinear.

23. For what value of 'k' is the following function continuous at x = 2?

$$f(x) = \begin{cases} 2x+1\,; x < 2 \\ k \quad ; x = 2 \\ 3x-1\,; x > 2 \end{cases}$$

24. Evaluate: $\begin{vmatrix} 2+\sqrt{3} & 3-\sqrt{2} \\ 3+\sqrt{2} & 2-\sqrt{3} \end{vmatrix}$

25. The radius of an air bubble is increasing at the rate of $\frac{1}{2}$ cm/s. At what rate is the volume of the bubble increasing when the radius is 1 cm?

OR

Show that the function f given by $f(x) = x^3 - 3x^2 + 4x$, $x \in R$ is increasing on R.

SECTION C
(Question numbers 26 to 31 carries 3 marks each)

26. Find $\int \frac{(x^2+x+1)}{(x+2)(x^2+1)}\, dx$

27. If $y = (tan^{-1}x)^2, show\ that\ (x^2+1)^2 y_2 + 2x\,(x^2+1)y_1 = 2$

OR

If $x^m\, y^n = (x+y)^{m+n}, prove\ that\ \frac{d^2y}{dx^2} = 0.$

28. Evaluate $\int_0^{\pi} \frac{x \tan x}{\sec x + \tan x}\, dx$

OR

Evaluate $\int_{-1}^{2} |x^3 - x|\, dx$

29. Find $\int x\, sin^{-1}x\ dx$

30. Find the value of $tan^{-1}\left(tan\frac{2\pi}{3}\right) + sin^{-1}\left(\sin\frac{2\pi}{3}\right)$

31. Find a vector which is perpendicular to both and $\vec{a}$ and $\vec{b}$ and $\vec{c}.\vec{d} = 15$.
where $\vec{a} = \hat{\imath} + 4\hat{\jmath} + 2\hat{k}, \vec{b} = 3\hat{\imath} - 2\hat{\jmath} + 7\hat{k}$ and $\vec{c} = 2\hat{\imath} - \hat{\jmath} + 4\hat{k}$.

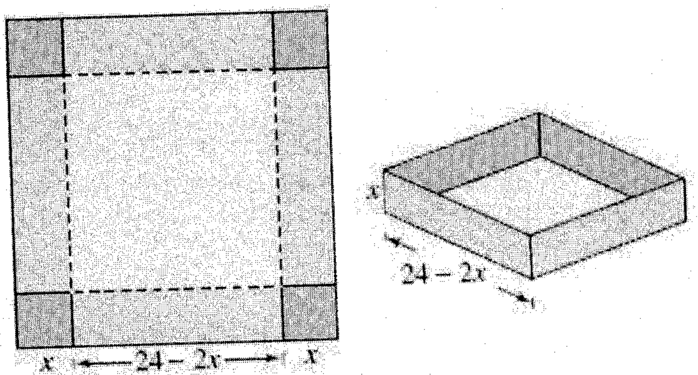
OR

The two skew lines L_1 and L_2 such that Line L_1 passes through the point A(1, 2, 1) and its direction is along the vector $\vec{b}_1 = \hat{\imath} - \hat{\jmath} + \hat{k}$, while Line L_2 passes through the point B(2, -1, -1) and its direction is along the vector $\vec{b}_2 = 2\hat{\imath} + \hat{\jmath} + 2\hat{k}$. Find the shortest distance between the two skew lines L_1 and L_2.

Section D

This section contains three Case-study based questions (from Q32 - Q34).

32. A man has an expensive square shape piece of golden board of size 24 cm. It is to be made into a box without top by cutting a square from each corner and folding the flaps to form a box.



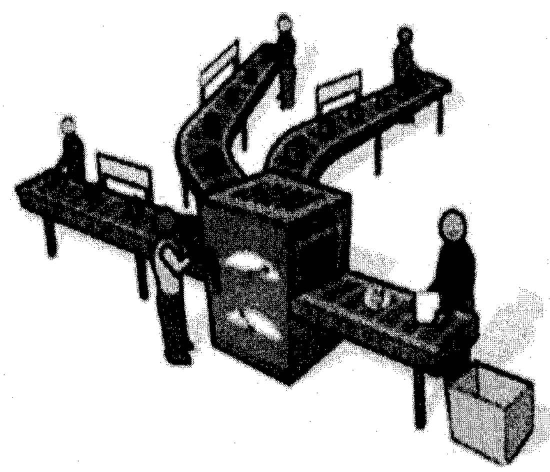
Based on the given information, answer the following questions.

(i) Write an expression for the volume of the open box.

(ii) In the first derivative test, if $f'(x)$ changes its sign from positive to negative as x increases through c, then function attains .... at c. (Fill in the blank)

(iii) What should be the side of the square piece to be cut off from each corner of the board so that the volume is maximum?

33. A factory produces three products every day. Their production on a particular day is 45 tons. It is found that production of third product exceeds the production of first product by 8 tons while the total production of first and third product is twice the production of second product. Let x, y and z be the production(in tons) of the first , second and the third product respectively. Based on the given information, answer the following questions.



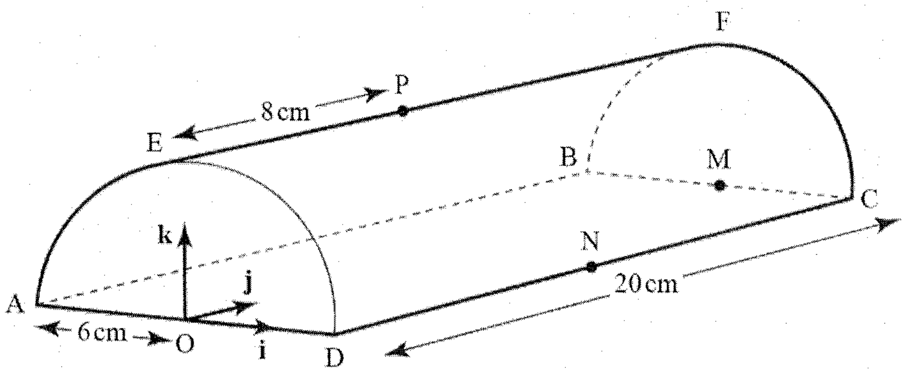
(i) Write the equations in terms of x, y and z and express it in the matrix form AX = B

(ii) Find A^{-1}

OR

(ii) How much is the production of each product?

34. The diagram shows a semi-circular prism with horizontal rectangular base ABCD. The vertical ends AED and BFC are semi-circles of radius 6 cm. The length of the prism is 20 cm. The mid-point of AD is the origin O, the midpoint of BC is M and the midpoint of DC is N. The points E and F are the highest points of the semi-circular ends of the prism. The point P lies on EF such that EP = 8 cm.



Coordinates of points A, D, P and M are (-6,0,0), (6,0,0), (0,8,6) and (0,20,0) respectively.
Unit vectors $\hat{\imath}, \hat{\jmath}$ *and* $\hat{k}$ are parallel to OD, OM and OE respectively.

Based on the given information, answer the following questions.
(i) Express each of the vectors $\overrightarrow{PA}$ and $\overrightarrow{PN}$ in terms of $\hat{\imath}, \hat{\jmath}$ *and* $\hat{k}$.
(ii) Calculate angle APN
(iii) Find a vector perpendicular to $\overrightarrow{PN}$ and $\overrightarrow{PM}$

OR

(iii) Find area of triangle MPN

Section E
(Question numbers 35 to 38 carries 5 marks each)

35. Find the area bounded by the curve $x^2 = 4y$ and the line $x = 4y - 2$.

OR

Using integration find the area of region bounded by the triangle whose vertices are (-1, 0), (1, 3) and (3, 2).

36. Solve the following linear programming problem graphically:
Maximise: $Z = 100x + 50y$ subject to the constraints: $x \geq 0, y \geq 0$,
$x + y \leq 300, 360x + 120y \leq 72000$ *and* $-x + y \leq 200$.
Also write the x and y coordinates of the point, at which Z is maximum.

37. Find the image of the point (1, 6, 3) in the line $\frac{x}{1} = \frac{y-1}{2} = \frac{z-2}{3}$

OR

Find the equation of the line passing through the point (-1, 3, -2) and perpendicular to each of the lines $\frac{x}{1} = \frac{y}{2} = \frac{z}{3}$ *and* $\frac{x+2}{-3} = \frac{y-1}{2} = \frac{z+1}{5}$.

38. Show that $R = \{(a, b): a, b \in A, |a - b| \text{ is divisible by } 4\}$ where $A = \{x \in Z : 0 \leq x \leq 12\}$ is an equivalence relation. Find the set of all elements related to 1. Also write the equivalence class [2].

****END OF THE QUESTION PAPER****